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# Dividing Device

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Applicant: Minsk Radio-Engineering Institute
Inventors: Alexander Nozik and Alexander Shostak
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*Abstract* — **This invention is related to the area of computing machinery and can be used to perform the division procedure. The invention significantly increases the performance of the known device for division by introducing two blocks for addition, a block for the formation of partial products and an inverter block. The division device allows for the beginning of the formation of redundant quotient digits simultaneously; it also converts the quotient to the binary form in parallel with the formation of the quotient redundant digits. The redundant quotient digits are determined from the equations obtained by assigning the sums of the respective partial products of the binary digits of the quotient and the divisor to the respective binary digits of the divisor.**

This invention is related to the area of computing machinery and can be used to perform the division procedure.

The purpose of the invention is to increase the performance of the known division device.

Fig. 1 provides the internal structure of the device. Fig. 2 provides the functional structure of the block for the formation of the partial products. Fig. 3-5 provide the functional structures of the first, second, and third addition blocks (for the case of $n = p = 6$, where $p$ is the number of redundant quotient digits and $n$ is the number of the binary digits of the dividend and the divisor).

The division device consists of block 1 that forms partial products, addition block 2 and addition block 3, the inverter block 4, the addition block 5, the divisor inputs $6_1$, the dividend inputs $6_2$, and the quotient outputs 7.

Block 1, which forms partial products (Fig.2), consists of 17 AND gates $8_1$ to $8_{17}$, and outputs $9_1$ to $9_{17}$.

The addition block 2 (Fig. 3) consists of six carry-save adders $10_1$ to $10_6$ and outputs $11_1$ to $11_6$ which provide the result in a redundant two-row format.

The addition block 3 (Fig. 4) consists of twelve carry-save adders $12_1$ – $12_{12}$ and outputs $13_1$ – $13_7$ which provide the result in a redundant two-row format.

The addition block 5 (Fig. 5) consists of ten carry-save adders $14_1$ to $14_{10}$ and carry look-ahead adder 15, the outputs of which are connected to the outputs 7 of the device.

The division device is based on the following idea. Assume that the dividend $C = 0.\ c_1\ c_2\ c_3\ c_4\ c_5\ c_6$ and the divisor $A = 0.\ a_1\ a_2\ a_3\ a_4\ a_5\ a_6$ are normalized binary numbers and the quotient $Q$ is represented as follows:

$$Q = C / A = q_0\ .\ q_1\ q_2\ q_3\ q_4\ q_5\ ,$$

where $q_i$ is $i$th redundant quotient digit and $0 \le i \le 5$.

The condition $A \cdot Q = C$ can be used to determine the redundant quotient digits by assigning the sums of the partial products with equal binary weights to the corresponding binary digits of the dividend $C$. As a result, the following system of linear equations can be obtained:

$$\begin{aligned}
&a_1q_0 = c_1\\
&a_1q_1 + a_2q_0 = c_2\\
&a_1q_2 + a_2q_1 + a_3q_0 = c_3\\
&a_1q_3 + a_2q_2 + a_3q_1 + a_4q_0 = c_4\\
&a_1q_4 + a_2q_3 + a_3q_2 + a_4q_1 + a_5q_0 = c_5\\
&a_1q_5 + a_2q_4 + a_3q_3 + a_4q_2 + a_5q_1 + a_6q_0 = c_6
\end{aligned} \qquad (1)$$

Taking into account the fact that $A$ and $C$ are normalized binary fractions i.e. $a_1 = c_1 = 1$, equations (1) can be rewritten as follows :

$$\begin{aligned}
&q_0 = 1\\
&q_1 = c_2 - a_2q_0\\
&q_2 = c_3 - a_2q_1 - a_3q_0\\
&q_3 = c_4 - a_2q_2 - a_3q_1 - a_4q_0\\
&q_4 = c_5 - a_2q_3 - a_3q_2 - a_4q_1 - a_5q_0\\
&q_4 = c_6 - a_2q_4 - a_3q_3 - a_4q_4 - a_5q_1 - a_6q_0
\end{aligned} \qquad (2)$$

Equations (2) can be solved by substituting the value of $q_0 = 1$ from the first equation to the second equation of (2). The resulting value of $q_1$ from the second equation along with the value of $q_0 = 1$ is substituted into the third equation. The resulting value of $q_2$ from the third equation of (2) along with the resulting value of $q_1$ and the value of $q_0 = 1$ are substituted to the fourth equation of (2) to obtain the resulting value of $q_3$. Continuing this procedure, the following non-recursive formulations for the redundant quotient digits $q_i$ $(0 \le i \le 5)$ can be obtained:

$$\begin{aligned}
&q_0 = 1\\
&q_1 = c_2 - a_2
\end{aligned}$$

$$q_2 = c_3 - a_3 - a_2c_2 + a_2$$
$$q_3 = c_4 - a_4 - a_3c_2 + 2a_2a_3 - a_2c_3 + a_2c_2 - a_2$$
$$q_4 = c_5 - a_5 - c_2a_4 + 2a_2a_4 - c_3a_3 + a_3 + 2c_2a_2a_3 - 3a_2a_3 - c_4a_2 + c_3a_2 + a_2 - c_2a_2$$

$$q_5 = c_6 - a_6 - c_2a_5 + 2a_2a_5 - c_3a_4 + 2a_3a_4 + 2c_2a_2a_4 - 3a_2a_4 - c_4a_3 + c_2a_3 + 2c_3a_2a_3 - 3c_2a_2a_3 - c_5a_2 + a_2a_3 + c_4a_2 - c_3a_2 - a_2 + c_2a_2 \quad (3)$$

Using $q_i$ $(0 \le i \le 5)$ from (3) , we will have the following formulation for $Q$:

$$Q = q_0 \cdot q_1 q_2 q_3 q_4 q_5 = q_0 2^0 + q_1 2^{-1} + q_2 2^{-2} + q_3 2^{-3} + q_4 2^{-4} + q_5 2^{-5} =$$
$$= 1 \cdot 2^0 + (c_2 - a_2)2^{-1} + (c_3 - a_3 - a_2c_2 + a_2)2^{-2} + (c_4 - a_4 - a_3c_2 + 2a_2a_3 - a_2c_3 + a_2c_2 - a_2)2^{-3} + (c_5 - a_5 - c_2a_4 + 2a_2a_4 - c_3a_3 + a_3 + 2c_2a_2a_3 - 3a_2a_3 - c_4a_2 + c_3a_2 + a_2 - c_2a_2)2^{-4} + (c_6 - a_6 - c_2a_5 + 2a_2a_5 - c_3a_4 + 2a_3a_4 + 2c_2a_2a_4 - 3a_2a_4 - c_4a_3 + c_2a_3 + 2c_3a_2a_3 - 3c_2a_2a_3 - c_5a_2 + a_2a_3 + c_4a_2 - c_3a_2 - a_2 + c_2a_2)2^{-5} \quad (4)$$

After grouping together the positive and the negative terms of (4), we will obtain the following :

$$Q = [\, 1 \cdot 2^0 + c_2 2^{-1} + (c_3 + a_2)2^{-2} + (c_4 + 2a_2a_3 + a_2c_2)2^{-3} + (c_5 + 2a_2a_4 + a_3 + 2c_2a_2a_3 + c_3a_2 + a_2)2^{-4} + (c_6 + 2a_2a_5 + 2a_3a_4 + 2c_2a_2a_4 + c_2a_3 + 2c_3a_2a_3 + a_2a_3 + c_4a_2 + c_2a_2)2^{-5} \,] -$$
$$- [a_2 2^{-1} + (a_3 + a_2c_2)2^{-2} + (a_4 + a_3c_2 + a_2c_3 + a_2)2^{-3} + (a_5 + c_2a_4 + c_3a_3 + 3a_2a_3 + c_4a_2 + c_2a_2)2^{-4} + (a_6 + c_2a_5 + c_3a_4 + 3a_2a_4 + c_4a_3 + 3c_2a_2a_3 + c_5a_2 + c_3a_2 + a_2)2^{-5} \,] \quad (5)$$

The terms in (5) that have the coefficients other than *1,* are grouped with the terms with different binary weights. For example, the positive term $2a_2a_3$ that has the binary weight $2^{-3}$ can be grouped with the positive terms with the binary weight $2^{-2}$ because $2a_2a_3 \cdot 2^{-3} = a_2a_3 \cdot 2^{-2}$ . For the same reason, the negative term $-3a_3a_2$ with the binary weight $2^{-4}$ can be substituted with the two negative terms $-a_3a_2$ with the binary weights equal to $2^{-3}$ and $2^{-4}$ respectively. After performing all the possible substitutions of the terms with the coefficients not equal to *1* in (5), we will have the following formulation for the quotient:

$$Q = [\, 1 \cdot 2^0 + c_2 2^{-1} + (c_3 + a_2 + a_2a_3)2^{-2} + (c_4 + a_2c_2 + a_2a_4 + c_2a_2a_3)2^{-3} + (c_5 + a_3 + c_3a_2 + a_2 + a_2a_5 + a_3a_4 + c_2a_2a_4 + c_3a_2a_3)2^{-4} + (c_6 + c_2a_3 +$$

$$+ a_2a_3 + c_4a_2 + c_2a_2)2^{-5} ] -$$
$$- [a_2 2^{-1} + ( a_3 + a_2c_2)2^{-2} + ( a_4 + a_3c_2 + a_2c_3 + a_2 + a_2a_3)2^{-3} + ( a_5 + c_2 a_4 +$$
$$+ c_3a_3 + a_2a_3 + c_4a_2 + c_2a_2 + a_2a_4 + c_2a_2a_3)2^{-4} + ( a_6 + c_2a_5 + c_3a_4 +$$
$$+ a_2a_4 + c_4a_3 + c_2a_2a_3 + c_5a_2 + c_3a_2 + a_2)2^{-5} ] \quad (6)$$

Equation (6) can be transformed into the algebraically equivalent form with a lesser number of terms. For example, the term $a_2$ is present in (6) with the positive sign and the binary weights equal to $2^{-2}$ and $2^{-4}$. It also appears in (6) with the negative sign and the binary weights equal to $2^{-1}$, $2^{-3}$, and $2^{-5}$. Since

$$a_2 2^{-2} + a_2 2^{-4} - a_2 2^{-1} - a_2 2^{-3} - a_2 2^{-5} = a_2 (2^{-2} + 2^{-4} - 2^{-1} - 2^{-3} - 2^{-5}) =$$
$$= a_2 (-2^{-2} - 2^{-4} - 2^{-5}) = a_2 (-2^{-2} - 2^{-3} + 2^{-5}) = a_2 2^{-5} - a_2 2^{-2} - a_2 2^{-3},$$

the term $a_2$, with the positive sign and the binary weights equal to $2^{-2}$ and $2^{-4}$ and the same term with the negative sign and the binary weights equal to $2^{-1}$, $2^{-3}$, and $2^{-5}$, can be substituted with the term $a_2$ with the positive sign and the binary weight equal to $2^{-5}$ and the negative sign and the binary weights equal to $2^{-2}$ and $2^{-3}$. That allows for the reduction of the number of terms $a_2$ from five to three therefore reducing the total number of terms in (6) by two. Using similar considerations, the term $a_2c_2$ in (6) with the positive sign and the binary weights equal to $2^{-3}$ and $2^{-5}$ and the negative sign with the binary weights $2^{-2}$ and $2^{-4}$ can be substituted with the same term with the negative sign and the binary weights equal to $2^{-3}$ and $2^{-5}$ . That allows for the reduction of the number of terms of $a_2c_2$ from four to two. After performing all the possible minimizations like the above, we will have the following formulation for the quotient:

$$Q = [ 1 \cdot 2^0 + c_2 2^{-1} + c_3 2^{-2} + c_4 2^{-3} + (c_5 + a_2a_5 + a_3a_4 + c_2a_2a_4 + c_3a_2a_3 + a_2a_3)2^{-4}$$
$$+ (c_6 + a_3a_2c_2 + a_2a_4 + a_2 + a_3 a_2)2^{-5} ] -$$
$$- [ a_2 2^{-2} + ( a_4 + a_3 + a_2 + a_2c_2)2^{-3} + ( a_5 + c_2 a_4 +$$
$$+ c_3a_3 + a_3 + c_3a_2 + c_2a_3)2^{-4} + ( a_6 + c_2a_5 + c_2a_3 + c_3a_4 +$$
$$+ a_2c_4 + c_4a_3 + c_5a_2 + c_3a_2 + a_2c_2)2^{-5} ] \quad (7)$$

The suggested division device uses equations (7) to formulate the quotient. Consider the case of dividing 6-bit normalized binary fractions. In that case, the binary digits $c_2$, $c_3$, $c_4$, $c_5$, and $c_6$ of the dividend and the binary digits $a_2$, $a_3$, $a_4$, $a_5$, and $a_6$ of the divisor are the inputs of the device. (The values of $c_1$ and $a_1$ are equal to $1$ and therefore are not provided explicitly as inputs). Block 1 formulates the values of the partial products according to (7). The values of the partial products that according to (7) have positive signs, are formulated on the outputs of block 1 that are connected to the second group of inputs of the addition block 2. The values of the partial products that according to (7) have negative signs, are formulated on the outputs of block 1 that are connected to the second group of inputs of the addition block 3. The first group of inputs of addition block 2 and addition block 3 are connected to the respective binary digits of the dividend and the divisor that are terms of (7). The addition block 2 and the addition block 3 work in parallel and their outputs are connected to the second and the third group of the inputs of

the addition block 5. However, the outputs of addition block 2 are connected to the corresponding inputs of addition block 5 directly and the outputs of addition block 3 are connected to the corresponding inputs of addition block 5 through the block of inverters 4. That allows for the generation of one's complement code based on the outputs of addition block 3. The generation of the two's complement code is implemented in addition block 5. The first group of inputs of addition block 5 is used to provide the inputs for the terms of (7) which have positive signs and are not connected to the inputs of addition block 2. The final result in non-redundant form $q_0^{'}.q_1^{'}q_2^{'}q_3^{'}q_4^{'}q_5^{'}$ is formulated on the outputs 7 of the device.

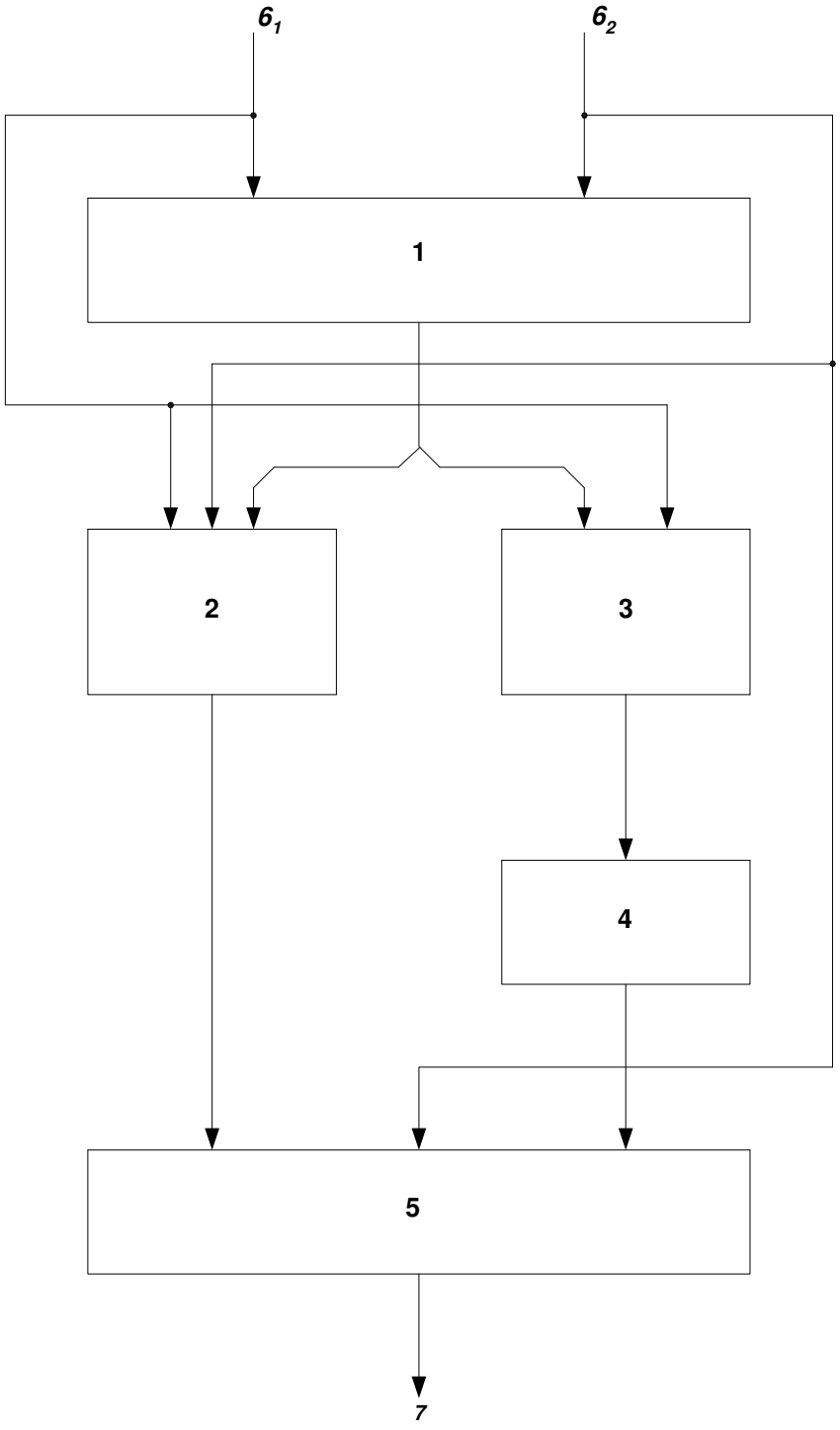
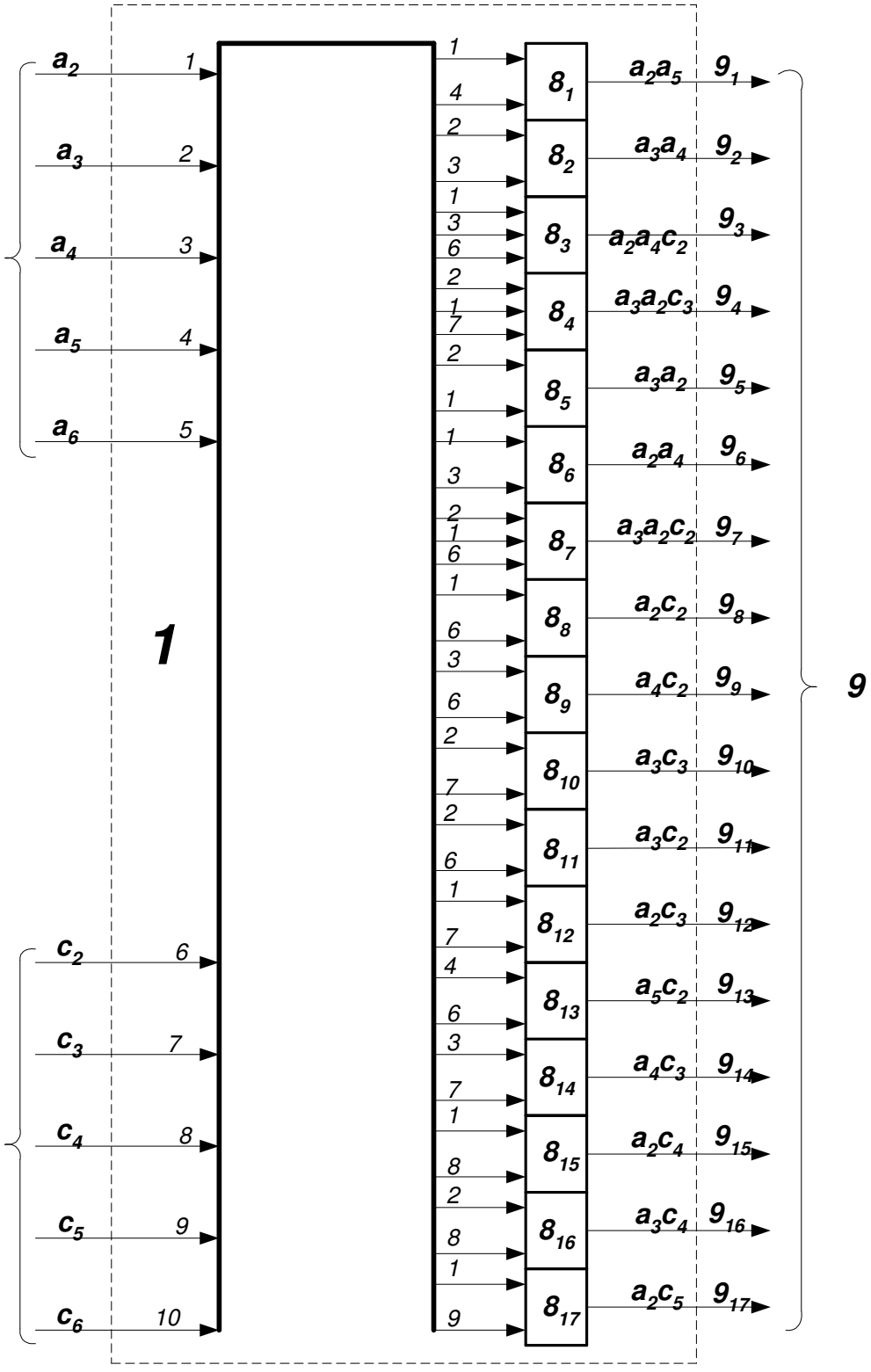



Fig. 1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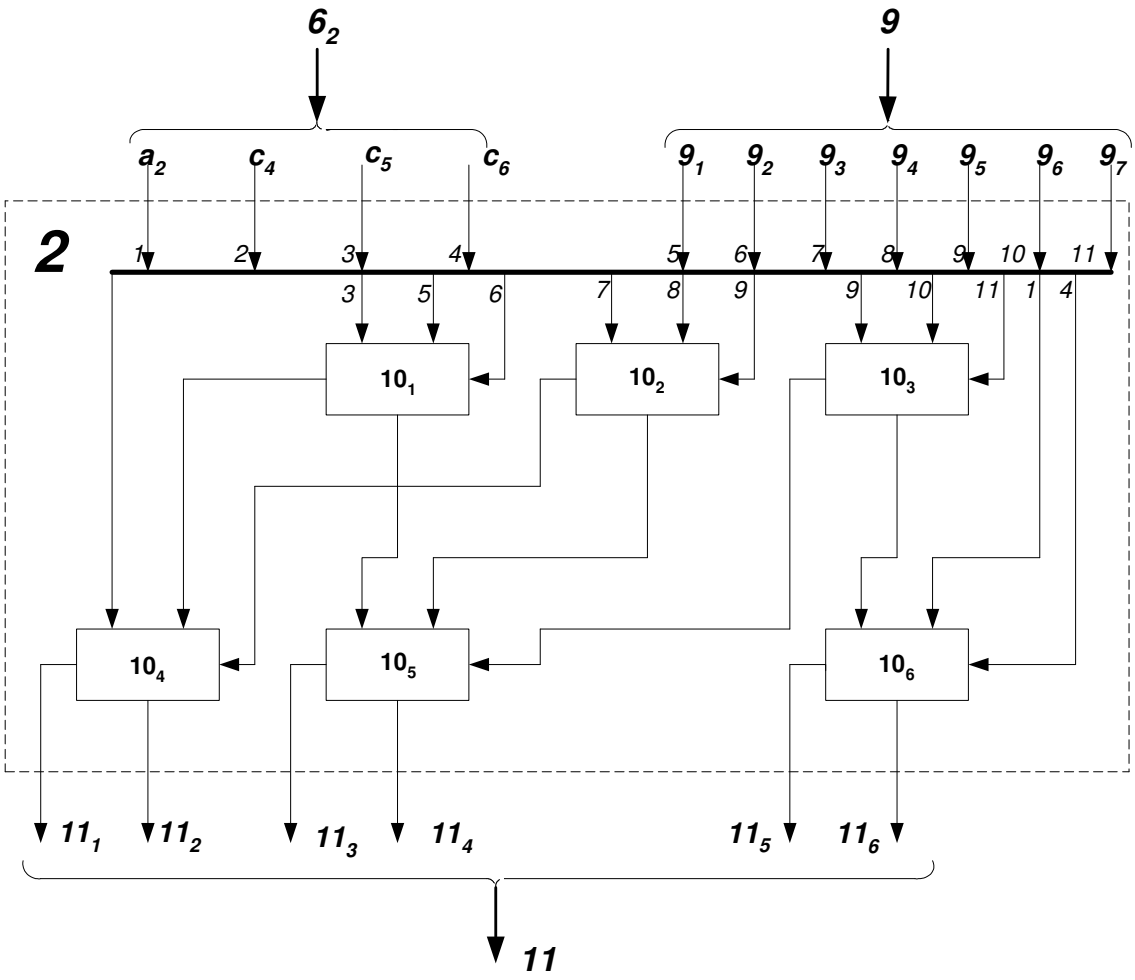
a3
a4
a5
a6
1
2
3
4
5
1
c2
c3
c4
c5
c6
6
7
8
9
10
1
8 1
8 2
8 3
8 4
8 5
8 6
8 7
8 8
8 9
8 10
8 11
8 12
8 13
8 14
8 15
8 16
8 17
a2a5 9 1
a3a4 9 2
a2a4c2 9 3
a3a2c3 9 4
a3a2 9 5
a2a4 9 6
a3a2c2 9 7
a2c2 9 8
a4c2 9 9
a3c3 9 10
a3c2 9 11
a2c3 9 12
a5c2 9 13
a4c3 9 14
a2c4 9 15
a3c4 9 16
a2c5 9 17
9

Fig. 2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9
a 2
c 4
c 5
c 6
9 1
9 2
9 3
9 4
9 5
9 6
9 7
2
1 2 3 4 5 6 7 8 9 10 11
3 5 6 7 8 9 9 10 11 1 4
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10 5
10 6
11 1
11 2
11 3
11 4
11 5
11 6
11

Fig. 3

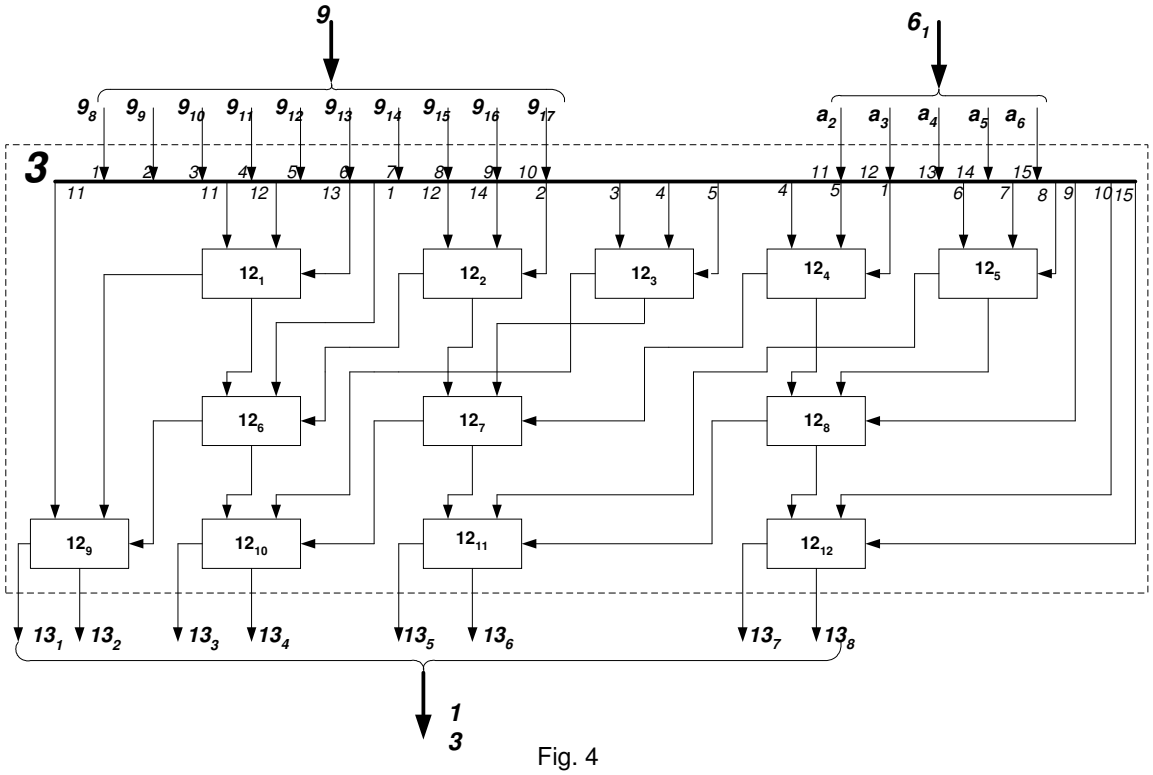
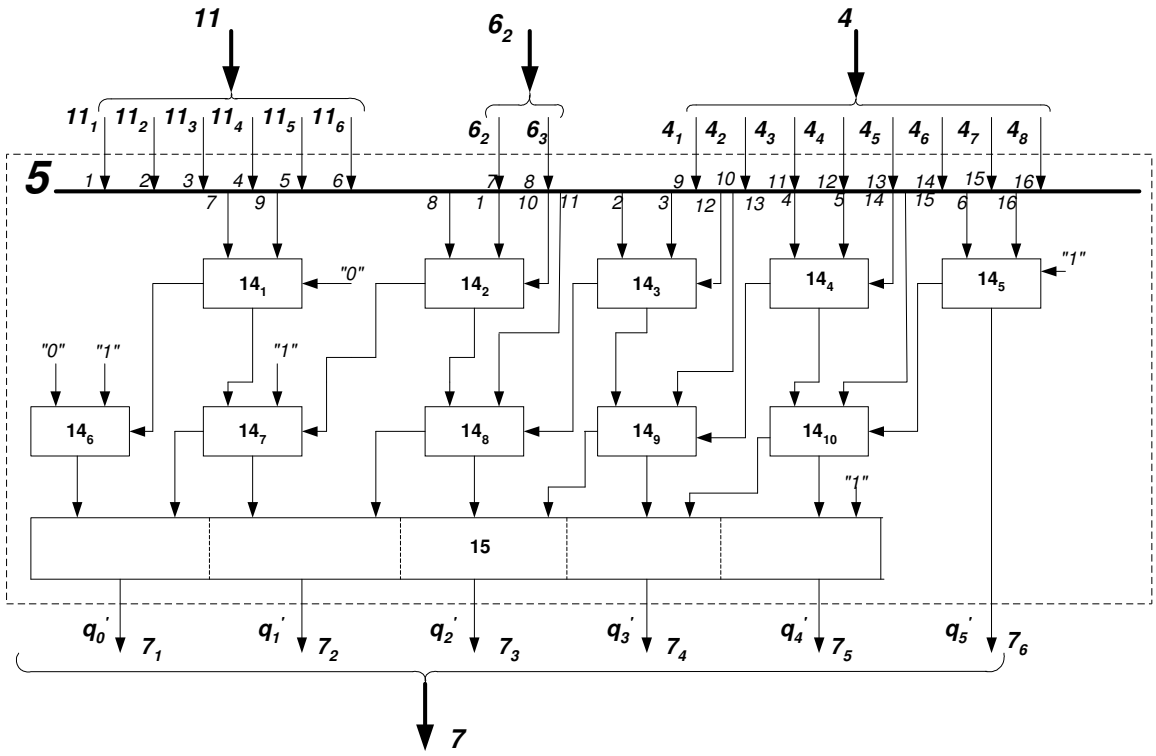

9
6 1
9 8 9 9 9 10 9 11 9 12 9 13 9 14 9 15 9 16 9 17
a 2 a 3 a 4 a 5 a 6
3
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
11 11 12 13 1 12 14 2 3 4 5 4 5 1 6 7 8 9 10 15
12 1 12 2 12 3 12 4 12 5
12 6 12 7 12 8
12 9 12 10 12 11 12 12
13 1 13 2 13 3 13 4 13 5 13 6 13 7 13 8
1
3

Fig. 4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4
11 1 11 2 11 3 11 4 11 5 11 6
6 2 6 3
4 1 4 2 4 3 4 4 4 5 4 6 4 7 4 8
5
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
7 9 8 1 10 11 2 3 12 13 4 5 14 15 6 16
14 1
"0"
14 2
14 3
14 4
14 5
"1"
"0" "1"
"1"
14 6
14 7
14 8
14 9
14 10
"1"
15
q0' 71 q1' 72 q2' 73 q3' 74 q4' 75 q5' 76
7

Fig. 5